2/18/20

Review of Notation

$$A \wedge B = \min(A, B)$$

$$(A)_+ = \begin{cases} A & \text{if } A \geq 0 \\ 0 & \text{if } A < 0 \end{cases}$$

R.V.'s

X = "ground-up loss" r.v.
= rvr dollar amount of loss before any policy modifications

= "severity" r.v.

Idea apply ~~mod~~ policy modifications (deductible, policy limit, coinsurance, etc.)

Then

Y^L = rvr insurance payment per loss

Y^P = rvr insurance payment per payment (not all loss result in a payment)

$$Y^P = Y^L \mid Y^L > 0$$

Policy Modifications

1) Ordinary deductible, d

Given X, ground-up loss r.v.

Then $Y^L = \begin{cases} 0 & \text{if } X \le d \\ X-d & \text{if } X > d \end{cases}$

$$\Rightarrow Y^L = (X-d)_+$$

$$\text{\&} \quad Y^P = X-d \mid X > d$$

Example: (Discrete) $\quad$ Given $d = 500$

Given

X	Pr
300	.4
900	.2
1000	.4

$\therefore$

Y^L	Pr
0	.4
400	.2
500	.4

Y^P	Pr
400	1/3
500	2/3

see below

$$Pr(Y^P = 400) = Pr(X = 900 \mid X > 500) = \frac{Pr(X=900 \wedge X>500)}{Pr(X>500)}$$

$$= \frac{Pr(X=900)}{Pr(X>500)} = \frac{.2}{.6} = \frac{1}{3}$$

$Pr(Y^P = 500) = 1 - \frac{1}{3} = \frac{2}{3}$ (or do the same as above)

Remark: $E[Y^L] = 280$

$$E[Y^P] = \frac{1400}{3} = 466.\overline{6}$$

Note: $Pr(X > d) = 0.6$ $\qquad 280 = 466.\overline{6}\,(0.6)$

$$\Longrightarrow \quad E[Y^L] = E[Y^P] \cdot Pr(X > d)$$

$$\boxed{\text{Fact: } \quad E[(Y^L)^k] \underset{\text{always}}{=} E[(Y^P)^k] \cdot Pr(X > d)}$$

Commonly Tested Examples

$$X \sim Exp(\theta) \Longrightarrow Y^P \sim Exp(\theta)$$

$$X \sim 2\text{-Pareto}(\alpha, \theta) \Longrightarrow Y^P \sim 2\text{-Pareto}(\alpha' = \alpha, \theta' = \theta + d)$$

$$X \sim U(0, \omega) \Longrightarrow Y^P \sim U(0, \omega - d)$$

2) Maximum Covered Amount, u

Given X

Then $Y^L = \begin{cases} X & \text{if } X \leq u \\ u & \text{if } X > u \end{cases}$

$$\Longrightarrow \quad Y^L = X \wedge u = Y^P$$

3) Ordinary deductible, d &
Maximum Covered Amount, u — Given X

$$Y^L = \begin{cases} 0 & \text{if } X < d \\ X - d & \text{if } d < X < u \\ \underbrace{u - d}_{\text{= policy limit = max amount paid by the insurer}} & \text{if } X > u \end{cases}$$

Note:
$$Y^L = \begin{cases} X - X & \text{if } X < d \\ X - d & \text{if } d < X < u \\ u - d & \text{if } X > u \end{cases}$$

$$\Rightarrow \quad Y^L = (X \wedge u) - (X \wedge d)$$

$$Y^P = Y^L \mid X > d$$

4) Coinsurance Factor, α — Given X

$$Y^L = \alpha \cdot X = Y^P$$

5) α, d, u — Given X

$$Y^L = \begin{cases} \alpha \cdot (X - X) & \text{if } X < d \\ \alpha(X - d) & \text{if } d < X < u \\ \underbrace{\alpha(u - d)}_{\text{= policy limit}} & \text{if } X > u \end{cases}$$

$$\therefore \quad Y^L = \alpha\left[(X \wedge u) - (X \wedge d)\right] \qquad Y^P = Y^L \mid X > d$$

6) Franchise Deductible, d Given X

$$Y^L = \begin{cases} 0 & \text{if } X < d \\ X & \text{if } X \geq d \end{cases}$$

$$Y^P = X \mid X > d \quad \left(= X - d \mid X > d \; + \; d \mid X > d \right)$$

$$\therefore \; E[Y^P_{Fr\,d}] = E[Y^P_{Ord\,d}] + d$$